

Non-local double-path Casimir phase shifts in atom interferometers

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We present a quantum open system theory of atom interferometers evolving in the quantized electromagnetic field bounded by an ideal conductor. Our treatment reveals an unprecedented feature of matter-wave propagation, namely the appearance of a non-local double-path phase coherence. Such a non-local phase arises from the coarse-graining over the quantized electromagnetic field and internal atomic degrees of freedom, yielding a non-Hamiltonian evolution of the atomic waves moving in presence of correlated quantum dipole and field fluctuations. We develop a diagrammatic interpretation of this phase, and estimate it for realistic experimental parameters.

Thanks to the advances in atom cooling achieved in the last decades, atom interferometry has become a field of great importance for both basic and applied science, enabling, in particular, the realization of extremely accurate inertial sensors [1, 2]. With the advent of the coherent atomic waves guided on chips [3], the investigation of atom-surface interactions has become a frontier for such systems. Already, atom interferometers have been used to experimentally probe the van der Waals regime [4]. This experimental effort calls for a complete theory of atom interferometers in the presence of quantum fluctuations of the electromagnetic (EM) field.

In this Letter, we layout such theory for a beam of neutral atoms, compute the phase shifts, and compare with the results obtained by standard techniques suitable for atoms driven by conservative forces. First, we develop a theory of the atomic phase-shifts taking the effect of field and atomic dipole fluctuations separately over each interferometer arm. This method, however, neglects quantum correlations, mediated by the field, between the atomic wavepackets evolving along the separate arms. In order to capture this new effect, we develop a theory of atom interferometers based on the influence functional method [5]. We show that non-local double-path phase shifts arise which, to our knowledge, have not been presented before. Such non-local phase is associated to pairs of paths rather than to individual ones. It is absent in the standard treatment of matter-wave dynamics with conservative forces, which shows that the effect of quantum vacuum and zero-point dipole fluctuations on atomic waves cannot be understood as an effective potential.

Typical atom interferometers [1, 2] involve the successive propagation of atomic samples through atom-optical elements such as atomic beam-splitters or atomic mirrors (usually implemented with the help of laser or magnetic fields) and in the dark (spatial regions with no applied fields). We consider here the phase arising from the propagation of atomic waves in the dark region along the two arms of the interferometer. We assume that the external atomic degrees of freedom (d.o.f.) evolve according to a quadratic and separable potential $V(\mathbf{r})$, which is a realistic situation in many atom optics experiments. The

initial atomic state is assumed Gaussian, i.e. $|e(t_0)\rangle = \int d^3\mathbf{r} \, wp(\mathbf{r}, \mathbf{r}_0, \mathbf{p}_0, \mathbf{w}_0)|\mathbf{r}\rangle$ with $wp(\mathbf{r}, \mathbf{r}_0, \mathbf{p}_0, \mathbf{w}_0) = \prod_{\eta=x,y,z} (1/\sqrt{\pi} w_{0\eta}) e^{-(\eta-\eta_0)^2/2w_{0\eta}^2 + i p_{0\eta}(\eta-\eta_0)/\hbar}$. If the atomic wave-packet is sufficiently dilute, the evolution is given by the *ABCD* theorem for atomic waves [6]:

$$\langle \mathbf{r}|e(t)\rangle = wp(\mathbf{r}, \mathbf{r}(t), \mathbf{p}(t), \mathbf{w}(t)) e^{\frac{i}{\hbar} S(\mathbf{r}_0, \mathbf{p}_0, t-t_0)} \quad (1)$$

for $t > t_0$. The average atomic position $\mathbf{r}(t)$ and momentum $\mathbf{p}(t)$ follow the classical equations of motion associated with the Hamiltonian \hat{H}_A , with the initial conditions $\mathbf{r}(t_0) = \mathbf{r}_0$ and $\mathbf{p}(t_0) = \mathbf{p}_0$. The *ABCD* approach, exact for non-interacting samples, can be extended to include moderate interactions [7] and atom lasers [8]. The width vector $\mathbf{w}(t)$, known analytically, is unimportant for the coming discussion. The action, $S(\mathbf{r}_0, \mathbf{p}_0, t-t_0)$, is obtained from the external and internal (energy $E(t)$) atomic Hamiltonian, integrated along the classical trajectory $S(\mathbf{r}_0, \mathbf{p}_0, t-t_0) = \int_{t_0}^t dt' \left(\frac{\mathbf{p}^2(t')}{2m} - E(t') - V(\mathbf{r}(t')) \right)$.

We begin with a local description of an atom interferometer in presence of quantum dipole and EM fluctuations. As depicted in the two diagrams of Fig. 1, we consider two arms sharing the same origin and corresponding to atoms flying between the instants t_0 and t_f above a metallic plate. One of the arms is parallel to the plate, and the other has a small velocity component normal to the plate. This simple configuration is sufficient for the discussion, which can be easily generalized to more complex interferometer geometries. The initial external atomic quantum state is a coherent superposition of two Gaussian wave-packets of common average position and different momenta, i.e. $|\psi_{e0}\rangle = \frac{1}{\sqrt{2}}|e_1(t_0)\rangle + |e_2(t_0)\rangle$ with $\langle \mathbf{r}|e_k(t_0)\rangle = wp(\mathbf{r}, \mathbf{r}_0, \mathbf{p}_{0k}, \mathbf{w}_0)$ as defined above. To account for the EM field fluctuations, we now consider the full Hamiltonian $\hat{H} = \hat{H}_A + \hat{H}_F + \hat{H}_{AF}$ with the field Hamiltonian $\hat{H}_F = \sum_{\mathbf{k}} \hbar \omega_{\mathbf{k}} \left(\hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}} + 1/2 \right)$ and the dipole interaction Hamiltonian $\hat{H}_{AF} = -\frac{1}{2} \hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\hat{\mathbf{r}}_a)$. Note that the dipole interaction involves three quantum operators, since the atomic position $\hat{\mathbf{r}}_a$ at which the electric field is

taken is also an observable.

We now apply the standard procedure in atom interferometry. Precisely, we assign to each interferometer path k a phase corresponding to the average dipole interaction energy for an external atomic state $|e_k\rangle$ following this path, i.e. $\phi_{st}^{(k)} = \frac{1}{4\hbar} \int_{t_0}^{t_f} dt \langle \hat{\mathbf{d}} \cdot \hat{\mathbf{E}}(\hat{\mathbf{r}}_a) + \hat{\mathbf{E}}(\hat{\mathbf{r}}_a) \cdot \hat{\mathbf{d}} \rangle_k(t)$. To derive this phase, we use linear response theory and treat the dipole interaction Hamiltonian H_{AF} as a perturbation [9]. In the Heisenberg picture, the atomic dipole operator then becomes the sum of a free-evolving contribution and of a contribution induced by the coupling to the on-atom electric field. The induced dipole operator is expressed as the convolution of the atomic polarizability with the free on-atom electric field playing the role of a source. By linearity of the coupling, the dipole and the on-atom electric field operators play symmetric roles, thus a similar result holds for the latter.

The phase $\phi_{st}^{(k)}$ is then expressed as a propagation integral involving two contributions, namely the response of the atomic dipole to the on-atom electric field fluctuations and the response of the on-atom electric field to the dipole fluctuations. The fluctuations of the dipole and electric field are captured by Hadamard Green's functions, whereas the susceptibilities (polarizability for the atom) correspond to retarded Green's functions. We assume that the width of the atomic wave-packet is small compared to the relevant EM field wavelengths, which allows us to replace the on-atom electric field operator at a given time by the electric field evaluated at the corresponding classical atomic position $\mathbf{r}_k(t)$ on the considered path k (electric dipole approximation). The retarded and Hadamard Green's functions read

$$\begin{aligned} G_{\hat{\mathbf{O}},ij}^R(x,x') &= \frac{i}{\hbar} \Theta(t-t') \langle [\hat{\mathcal{O}}_i^f(x), \hat{\mathcal{O}}_j^f(x')] \rangle \\ G_{\hat{\mathbf{O}},ij}^H(x,x') &= \frac{1}{\hbar} \langle \{ \hat{\mathcal{O}}_i^f(x), \hat{\mathcal{O}}_j^f(x') \} \rangle \end{aligned} \quad (2)$$

with the operator $\hat{\mathbf{O}} = \hat{\mathbf{d}}, \hat{\mathbf{E}}$, where $\hat{\mathcal{O}}^f$ corresponds to the free-evolving Heisenberg operator. For the dipole operator $\hat{\mathbf{O}} = \hat{\mathbf{d}}$, the arguments of the Green's functions are two instants $(x, x') = (t, t')$. For the electric-field operator $\hat{\mathbf{O}} = \hat{\mathbf{E}}$, these arguments are two four-vectors $(x, x') = (\mathbf{r}, t, \mathbf{r}', t')$. Last, we use the isotropy of the dipole Green's functions to write the standard Casimir phase as:

$$\begin{aligned} \phi_{st}^{(k)} &\simeq \frac{1}{4} \int_{t_0}^{t_f} \int_{t_0}^{t_f} dt dt' \left[G_{\hat{\mathbf{d}}}^H(t, t') \text{Tr} G_{\hat{\mathbf{E}}}^R(r_k(t), r_k(t')) \right. \\ &\quad \left. + G_{\hat{\mathbf{d}}}^R(t, t') \text{Tr} G_{\hat{\mathbf{E}}}^H(r_k(t), r_k(t')) \right] \end{aligned} \quad (3)$$

with the four-vectors $r_k(t) = (\mathbf{r}_k(t), t)$ and Tr denoting the trace of the Green's functions $G_{\hat{\mathbf{E}},ij}^{R,H}$ with respect to its tensorial components. We note that this phase already contains non-trivial dynamical Casimir effects [10], beyond the plain integration of the Casimir potential taken

at the instantaneous position along the classical trajectory, particularly in the case of a time-dependent atom-surface distance as in the trajectory $\mathbf{r}_2(t)$ shown in Fig. 1.

We now turn to a rigorous computation of the phase with closed-time-path (CTP) integrals [5]. Such path integrals involve simultaneously forward and backward histories of the system. The coarse-graining over the quantum field induces a mixing between these two histories, which will be associated to a pair of interferometer paths. This is at the origin of a path entanglement inducing the double-path phase discussed below. First, we specify the actions for the considered quantum d.o.f., namely $S_E[\mathbf{r}_a] = \int_{t_i}^{t_f} dt \left(\frac{M}{2} \dot{\mathbf{r}}_a^2(t) - V[\mathbf{r}_a(t)] \right)$ for the atomic position \mathbf{r}_a , $S_F[A^\mu] = (\epsilon_0/4) \int d^4x F^{\mu\nu} F_{\mu\nu}$ with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ for the EM field, and the action for the dipolar interaction $S_{AF}[A_\mu, \mathbf{d}, \mathbf{r}_a] = - \int d^4x J^\mu(x) A_\mu(x)$ defined with the current $J^\mu[\mathbf{d}, \mathbf{r}_a](x) = - \int dt d_i(t) \kappa_i^\mu \delta^{(4)}(x - r_a(t))$. The four-dimensional integrals are defined as $\int d^4x = \int_{t_0}^{t_f} dt \int d^3\mathbf{r}$. We have introduced the differential operator $\kappa_{i\mu} = \partial_i \eta_{0\mu} - \partial_0 \eta_{i\mu}$ ($\eta_{\mu\nu}$ is the Minkowski metric with mostly plus signature) relating the electric field to the vector potential by contraction: $E_i(x) = \kappa_i^\mu A_\mu(x)$. The following discussion is valid for an arbitrary action $S_A[\mathbf{d}]$ for the internal atomic d.o.f.. We consider the evolution of the density matrix $\rho(\mathbf{r}_a, \mathbf{d}, A^\mu, t)$ and trace over the field and dipole d.o.f.. The result can be expressed as a CTP integral over the position involving an influence action $S_{IF}[\mathbf{r}_a, \mathbf{r}'_a]$

$$\rho(\mathbf{r}_f, \mathbf{r}'_f, t) = \int_{\text{CTP}}^{\mathbf{r}_f, \mathbf{r}'_f} \mathcal{D}\mathbf{r}_a e^{\frac{i}{\hbar} (S_E[\mathbf{r}_a] - S_E[\mathbf{r}'_a] + S_{IF}[\mathbf{r}_a, \mathbf{r}'_a])} \langle 4 \rangle$$

where \mathbf{r}_a and \mathbf{r}'_a refer to forward and backward histories, respectively. We have noted for later convenience the CTP integral for a generic d.o.f. X :

$$\int_{\text{CTP}}^{X_f, X'_f} \mathcal{D}X = \iint dX_0 dX'_0 \int_{X_0}^{X_f} \mathcal{D}X \int_{X'_0}^{X'_f} \mathcal{D}X' \rho(X_0, X'_0, t_0) \quad (5)$$

with $\rho(t_0)$ the initial density matrix. At this level, the influence action $S_{IF}[\mathbf{r}_a, \mathbf{r}'_a]$ fully accounts for the effects of the field and the internal atomic dynamics on the external atomic d.o.f.. The final density matrix $\rho(t_f)$ contains four sharp peaks in the region $(\mathbf{r}_a, \mathbf{r}'_a)$ centered around the classical positions $(\mathbf{r}_i(t_f), \mathbf{r}_j(t_f))$ for $i, j = 1, 2$. The desired phase shift is obtained from the off-diagonal density matrix elements (i.e. $i \neq j$). The main contribution to the path integral comes from the paths in the vicinity of the two stationary paths. Thus, the phase difference due to quantum fluctuations can be evaluated on the classical paths by taking the real part of the influence action $\phi_{if} = \frac{1}{\hbar} \text{Re} [S_{IF}[\mathbf{r}_2, \mathbf{r}_1]]$ [11] (whereas the imaginary part represents the decoherence due to the plate [12]). To derive these classical paths, one can neglect the influence action corresponding to the Casimir-Polder interaction, which amounts to finding the Casimir phase shifts by considering undeflected trajectories.

Let us detail the procedure to obtain the influence action in the spirit of previous derivations of non-equilibrium forces mediated by a quantum field [10, 13]. First, we define an intermediate influence action $S_{IF}^{A\mu}[\mathbf{d}, \mathbf{d}', \mathbf{r}_a, \mathbf{r}'_a]$ corresponding to the trace over the EM field alone:

$$e^{\frac{i}{\hbar} S_{IF}^{A\mu}[\mathbf{d}, \mathbf{d}', \mathbf{r}_a, \mathbf{r}'_a]} = \int \mathcal{D}A_{\mu}^f \int_{\text{CTP}}^{A_{\mu}^f, A_{\mu}^f} \mathcal{D}A_{\mu} \times e^{\frac{i}{\hbar} (S_F[A_{\mu}] + S_{AF}[A_{\mu}, \mathbf{d}, \mathbf{r}_a] - S_F[A'_{\mu}] - S_{AF}[A'_{\mu}, \mathbf{d}', \mathbf{r}'_a])} \quad (6)$$

This CTP integral over the vector potential linearly coupled to an external current is well-known (see for instance Ref. [11])

$$S_{IF}^{A\mu}[\mathbf{d}, \mathbf{d}', \mathbf{r}_a, \mathbf{r}'_a] = \iint d^4x d^4x' \left[J^{\mu-}(x) G_{\hat{A}, \mu\nu}^R(x, x') J^{\nu+}(x') + \frac{i}{4} J^{\mu-}(x) G_{\hat{A}, \mu\nu}^H(x, x') J^{\nu-}(x') \right] \quad (7)$$

Taking standard conventions, we have introduced the semi-sum $J^+ = \frac{1}{2}J[\mathbf{d}, \mathbf{r}_a] + \frac{1}{2}J[\mathbf{d}', \mathbf{r}'_a]$ and difference $J^- = J[\mathbf{d}, \mathbf{r}_a] - J[\mathbf{d}', \mathbf{r}'_a]$ variables. The vector potential's retarded and Hadamard Green's functions $G_{\hat{A}, \mu\nu}^{R, H}$ are defined as the electric field Green's functions $G_{\hat{\mathbf{E}}, ij}^H$ in Eq. (2) with Cartesian coordinates replaced by Lorentz indices.

To obtain the desired influence action $S_{IF}[\mathbf{r}_a, \mathbf{r}'_a]$, we average the EM influence functional given by Eqs. (6, 7): $e^{\frac{i}{\hbar} S_{IF}[\mathbf{r}_a, \mathbf{r}'_a]} = \langle e^{\frac{i}{\hbar} S_{IF}^{A\mu}[\mathbf{d}, \mathbf{d}', \mathbf{r}_a, \mathbf{r}'_a]} \rangle_{\mathbf{d}}$ with $\langle \dots \rangle_{\mathbf{d}}$ denoting the time-dependent average over the free-evolving dipole d.o.f.. We take an approximation of small dipolar coupling, namely $e^{\frac{i}{\hbar} S} = 1 + \frac{i}{\hbar} S + O(d^3)$ for the actions S_{IF} and $S_{IF}^{A\mu}$. Thus the influence action reads $S_{IF}[\mathbf{r}_a, \mathbf{r}'_a] \simeq \langle S_{IF}^{A\mu}[\mathbf{d}, \mathbf{d}', \mathbf{r}_a, \mathbf{r}'_a] \rangle_{\mathbf{d}}$. The vector potential Green's functions can be treated as constants when averaging over the dipole d.o.f.. In order to express the influence action in terms of EM and dipole correlation functions, we expand the currents J^{\pm} in terms of $J[\mathbf{d}, \mathbf{r}_a]$ and $J[\mathbf{d}', \mathbf{r}'_a]$ in Eq. (7) and integrate over the spatial coordinates. The influence action receives a single-path (SP) and a double-path (DP) contribution, i.e. $S_{IF}[\mathbf{r}_2, \mathbf{r}_1] = S_{IF}^{\text{SP}}[\mathbf{r}_2, \mathbf{r}_1] + S_{IF}^{\text{DP}}[\mathbf{r}_2, \mathbf{r}_1]$ with

$$\begin{aligned} S_{IF}^{\text{SP}}[\mathbf{r}_2, \mathbf{r}_1] &= \frac{\hbar}{2} \iint_{t_0}^{t_f} dt dt' \left[g_{t, t'}^F \text{Tr} G_{\hat{\mathbf{E}}}^R(r_2(t), r_2(t')) - g_{t, t'}^{F*} \text{Tr} G_{\hat{\mathbf{E}}}^R(r_1(t), r_1(t')) \right] \\ &+ \frac{i}{2} \left[g_{t, t'}^F \text{Tr} G_{\hat{\mathbf{E}}}^H(r_2(t), r_2(t')) + g_{t, t'}^{F*} \text{Tr} G_{\hat{\mathbf{E}}}^H(r_1(t), r_1(t')) \right] \\ S_{IF}^{\text{DP}}[\mathbf{r}_2, \mathbf{r}_1] &= \frac{\hbar}{2} \iint_{t_0}^{t_f} dt dt' \left[g_{t, t'}^+ \text{Tr} G_{\hat{\mathbf{E}}}^R(r_2(t), r_1(t')) - g_{t, t'}^{+*} \text{Tr} G_{\hat{\mathbf{E}}}^R(r_1(t), r_2(t')) \right. \\ &\left. - \frac{i}{2} \left[g_{t, t'}^+ \text{Tr} G_{\hat{\mathbf{E}}}^H(r_2(t), r_1(t')) + g_{t, t'}^{+*} \text{Tr} G_{\hat{\mathbf{E}}}^H(r_1(t), r_2(t')) \right] \right] \quad (8) \end{aligned}$$

We have introduced the plain and time-ordered correlations of a free-evolving single dipole component (all diagonal components are equivalent by isotropy), respectively $g_{t, t'}^+ = \frac{1}{\hbar} \langle \hat{d}_i(t) \hat{d}_i(t') \rangle_{\mathbf{d}}$ and $g_{t, t'}^F = \frac{1}{\hbar} \langle T \hat{d}_i(t) \hat{d}_i(t') \rangle_{\mathbf{d}}$. $\phi_{if}^{\text{SP}} = \frac{1}{\hbar} \text{Re} [S_{IF}^{\text{SP}}[\mathbf{r}_2, \mathbf{r}_1]]$ and $\phi_{if}^{\text{DP}} = \frac{1}{\hbar} \text{Re} [S_{IF}^{\text{DP}}[\mathbf{r}_2, \mathbf{r}_1]]$ are the single and double path influence phases respectively. To evaluate the single-path phase ϕ_{if}^{SP} , we note that both electric field Green's functions are real and use the general dipole relations $\text{Re} [g_{t, t'}^F] = \frac{1}{2} G_d^H(t, t')$ and $\text{Im} [g_{t, t'}^F] = -\frac{1}{2} (G_d^R(t, t') + G_d^R(t', t))$. One then obtains that $\phi_{if}^{\text{SP}} = \phi_{st}^{(2)} - \phi_{st}^{(1)}$, namely the SP influence phase coincides exactly with the standard local phase of Eq. (3).

On the other hand, the presence of a non-local double-path phase ϕ_{if}^{DP} contrasts sharply with the local phase obtained by a standard atom interferometric method. In order to interpret and evaluate this phase, a first step is to derive the trace of the electric field Green's functions [Eq. (2)] in the presence of a perfect infinite plane conductor placed at $z = 0$. The result can be written as the sum of a free-space and of a scattered contribution, namely $\text{Tr} G_{\hat{\mathbf{E}}}^R(r, r') = \mathcal{G}_{\hat{\mathbf{E}}}^{R, 0}(r, r') + \mathcal{G}_{\hat{\mathbf{E}}}^{R, S}(r, r')$ with

$$\begin{aligned} \mathcal{G}_{\hat{\mathbf{E}}}^{R, 0}(r, r') &= \frac{\Theta(\tau)}{2\pi\epsilon_0 c^2} \frac{\partial^2}{\partial \tau^2} \left(\frac{\delta(\tau - |\mathbf{R}|/c)}{|\mathbf{R}|} \right) \\ \mathcal{G}_{\hat{\mathbf{E}}}^{R, S}(r, r') &= \frac{\Theta(\tau)}{2\pi\epsilon_0} \frac{\partial^2}{\partial z \partial z'} \left(\frac{\delta(\tau - |\mathbf{R}_I|/c)}{|\mathbf{R}_I|} \right) \quad (9) \end{aligned}$$

By translational invariance, the free-space contribution $\mathcal{G}_{\hat{\mathbf{E}}}^{R, 0}(r, r')$ depends only on the relative position $\mathbf{R} = \mathbf{r} - \mathbf{r}'$. The scattered contribution involves the image of the dipole, and thus we have introduced $\mathbf{R}_I = \mathbf{r} - \mathbf{r}'_I$ with the image position $\mathbf{r}'_I = (x', y', -z')$. Both contributions depend on the time difference $\tau = t - t'$ and not on the individual times. A similar expression holds for the Hadamard Green's function, but the corresponding phase contribution oscillates so fast that it can be neglected in practice.

The influence action of Eq. (8) appears as a sum of propagation integrals, which can be turned easily into Feynman-like diagrams involving the simultaneous propagation on distinct paths. Using the Green's functions of Eq. (9), one obtains the DP phase as the difference of two free-space diagrams (a)–(b) plus the difference between two scattered diagrams (c)–(d) as depicted in Fig. 1. Each diagram involves an advanced time t on one path and a retarded time t' on the other path (for the free-space diagrams) or on the image of the other path (for the scattered diagrams). These paths will be denoted respectively the advanced (red arrow) and the retarded path (blue arrow). The purple arrow stands for the free-space (diagrams (a) and (b)) or for the scattered (diagrams (c) and (d)) retarded electric field Green's functions. These diagrams show that quantum dipole fluc-

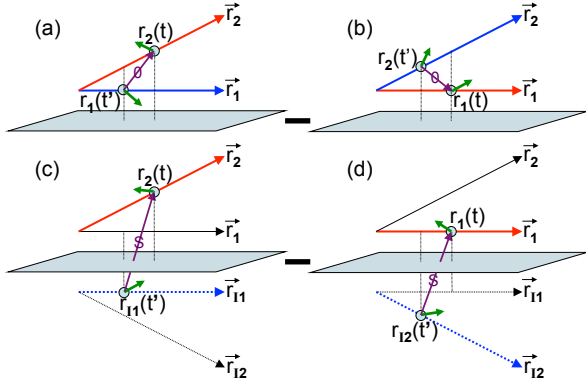


FIG. 1: (Color online) Diagrammatic expression of the DP phase. The delay $t - t'$ between the advanced and retarded times corresponds to the propagation of light over the distances $|\mathbf{r}_l(t') - \mathbf{r}_m(t)|$ (for the diagrams (a,b)) and $|\mathbf{r}_{ll}(t') - \mathbf{r}_m(t)|$ (for the diagrams (c,d)) with $l, m = 1, 2, l \neq m$.

tuations propagate, via the quantum electric field, from the retarded path (four-position $\mathbf{r}_l(t')$ for the free-space diagrams, image four-position $\mathbf{r}_{ll}(t')$ for the scattered diagrams (c,d)) to the advanced path four-position $\mathbf{r}_m(t)$ with $m \neq l$. Such diagrams express the Casimir interaction between two coherent components of a quantum state associated with a single atom, travelling on different arms (free-space diagrams), or on one arm and on the image of the other arm (scattered diagrams).

Let us focus on the scattered diagrams (c,d). Since the instants t, t' are constrained by the causality principle ($t' < t$) and by the speed of light, the atom moves on the advanced arm during the propagation time $t - t' \simeq |\mathbf{r}_1(t) - \mathbf{r}_{l2}(t)|/c$ from the retarded path to the advanced path. The resulting distance is different depending on whether the path 1 or the path 2 is the advanced path, because they correspond to different velocities. This yields an asymmetry between diagrams (c) and (d): the distance between the retarded image position $\mathbf{r}_{ll}(t')$ and the advanced position $\mathbf{r}_m(t)$ is indeed slightly larger in the left diagram (c) than in the right diagram (d), thereby yielding a stronger cross-talk between the quantum dipole fluctuations on distinct paths in the latter. The DP phase is essentially a signature of this asymmetry. Note that such asymmetry is reminiscent of two basic features of the EM interaction, namely the causality and the finite speed of the propagation. Such asymmetry arises also in the free-space diagrams (a) and (b), precisely the quantum dipole cross-talk is always weaker on the diagram attributing the advanced times to the faster path. In the short-distance limit considered below, this free-space contribution is actually of much smaller amplitude, and independent of the distance z_0 . It is nevertheless remarkable that a non-local DP phase persists even in empty space. This effect will be discussed in detail elsewhere.

We now give an analytical expression of the double-

path phase for a realistic experimental system, taking the two classical paths shown in Fig. 1 expressed as $\mathbf{r}_1(t) = \mathbf{r}_0 + v_{\parallel}(t-t_0)\hat{\mathbf{x}}$ and $\mathbf{r}_2(t) = \mathbf{r}_0 + v_{\parallel}(t-t_0)\hat{\mathbf{x}} + v_{\perp}(t-t_0)\hat{\mathbf{z}}$. To evaluate the dipole correlation functions $g_{t,t'}^+$, we modeled the dipole as an harmonic oscillator with a transition frequency ω_0 , which yields the correlation function $g_{t,t'}^+ = \frac{1}{2}\alpha(0)\omega_0 e^{-i\omega_0(t-t')}$ expressed as a function of the static atomic polarizability $\alpha(0)$. In the short-distance limit $z_0 \ll \lambda_0$, the DP phase reads ($T = t_f - t_0$)

$$\phi_{if}^{\text{DP}} = \frac{3\pi}{4\lambda_0} \left(\frac{\alpha(0)}{4\pi\epsilon_0} \right) \left(\frac{1}{z_0^2} - \frac{1}{(z_0 + v_{\perp}T)^2} \right) \quad (10)$$

where $\lambda_0 = 2\pi c/\omega_0$ is the transition wavelength. For a short path separation $v_{\perp}T \ll z_0$, the phase ϕ_{if}^{DP} becomes proportional to the endpoint separation $v_{\perp}T$. For a long path separation, i.e. $z_0 \ll v_{\perp}T \ll c/\omega_0$, the DP phase saturates to a maximal value depending only on the atomic polarizability, on the transition wavelength and on the altitude z_0 . We have investigated this value for a sample of ^{87}Rb atoms with a static polarizability $\alpha_{\text{Rb}}(0)/(4\pi\epsilon_0) = 4.72 \times 10^{-29} \text{ m}^3$ where the main transition has a wavelength $\lambda_0 \simeq 790 \text{ nm}$. We have considered a distance to the plate of $z_0 = 20 \text{ nm}$ similar to the distance used in the experiments of Ref. [4]. Such parameters yield a DP phase of $\phi_{if}^{\text{DP}} = 3.5 \times 10^{-7} \text{ rad}$. This value is beyond the sensitivity given by the state of the art atom interferometers but still larger than systematic phases considered in atom gravimeters [2].

To conclude, we have developed a quantum open system theory of atom interferometers, predicting non-local double-path phase shifts in the propagation of atomic waves. We have shown that the standard atom-optics approach catches only the local phase shifts corresponding to single-path terms obtained with the influence functional method. Such non-local DP phase is a consequence of the coarse-graining over the EM field, which induces cross talks between quantum dipole fluctuations on each arm. The DP phase constitutes a one-particle quantum interference effect, which exists thanks to the coherence of the matter waves propagating in the interferometer. We have developed a diagrammatic picture of this effect, which occurs even in free space, and can be interpreted as an asymmetry between diagrams involving simultaneous atomic propagation on distinct paths. This asymmetry can be seen as a dynamical relativistic correction of the quantum dipole cross-talks between the interferometer arms. We have shown that the DP phase shift compares to systematics considered for accurate atom interferometers. Our approach can be extended to multiple-path atom interferometers by considering pairs of paths. This is to our knowledge the first evidence of a non-local phase coherence in atom optics.

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